两个力学问题 (10 分)

开始题目之前请仔细阅阅每个信封中的考试指南。

A 部分. 隐藏的盘子 (3.5 分)

一个半径为 r_1 厚 w_1 的实心木质圆柱，里面固定一个半径为 r_2，厚 w_2 圆柱形金属盘。金属盘对称轴为 B，木质圆柱对称轴为 S，金属盘与木质圆柱同轴放置，S 与 B 之间的距离为 d；金属盘与木质圆柱两底面等距离。木质圆柱的密度为 ρ_1, 金属的密度 $\rho_2 > \rho_1$。木质圆柱与金属盘的总质量为 M。

In this task, we place the cylinder on the ground so that it can freely roll to the left and right. See Fig. 1 for a side view and a view from the top of the setup.

在此问中，将木质圆柱横向水平放置在地面上，木质圆柱可以左右自由滚动，见图 1. 侧视图与俯视图。

The goal of this task is to determine the size and the position of the metal disk.

任务的目标是确定金属盘的尺寸 r_2 和 w_2 以及在木质圆柱中的位置 d。

In what follows, when asked to express the result in terms of known quantities, you may always assume that the following are known:

在下面的问题中用已知物理量表示计算结果，并始终假定以下参量为已知，

$$r_1,\ w_1,\ \rho_1,\ \rho_2, M\ . \tag{1}$$

The goal is to determine r2,w2 and d, through indirect measurements.

目标是确定金属盘的尺寸 r_2 和 w_2 以及在木质圆柱中的位置，非直接测量。

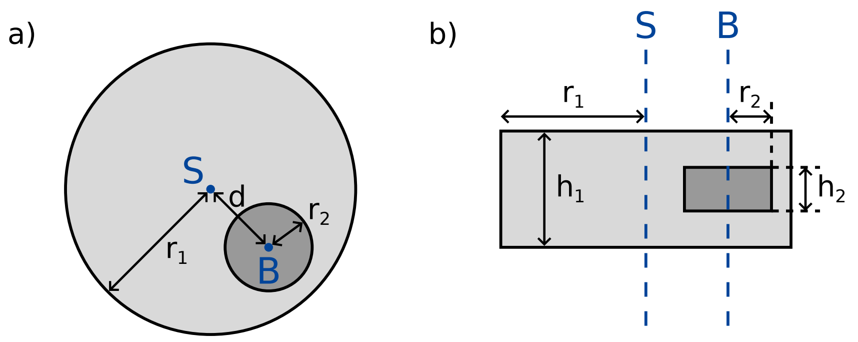

Figure 1: a) side view b) view from top
图 1：a) 侧视图；b) 俯视图

b is the distance between the centre of mass C of the whole system and the symmetry axis S of the cylinder. In order to determine this distance, we design the following experiment: We place the cylinder on a horizontal base in such a way that it is in a stable equilibrium. Let us now slowly incline the base by an angle (see Fig. 2). As a result of the static friction, the cylinder can roll freely without sliding. It will roll down the incline a little bit, but then come to rest in a stable equilibrium after rotating by an angle which we measure.

b 为系统质心 C 与木质圆柱对称轴 S 之间的距离。为确定 b，我们设计了如下实验：将系统放置在一个水平基座上，呈稳定平衡。缓慢倾斜基座使之与地面成 Θ 角 (见图 2)。由于静摩擦力 圆柱与斜面间无滑动滚动。当圆柱沿斜面滚动一小段距离后，再次进入一个稳定平衡状态，测得平衡时圆柱转过的角度为 ϕ。

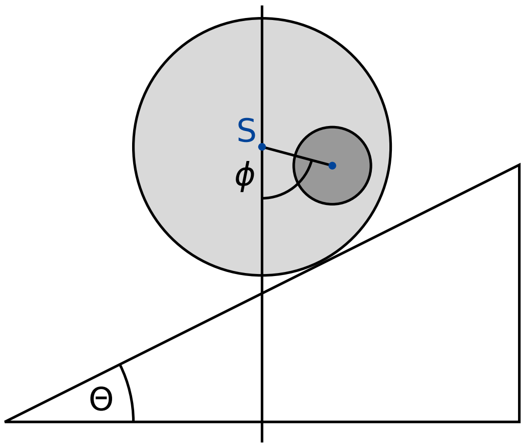

Figure 2: Cylinder on an inclined base.
图 2. 倾斜的基座及圆柱

A.1	Find an expression for b as a function of the quantities (1), the angle and the tilting angle of the base. 给出 b 的表达式，用 (1) 中的参量以及 ϕ 和 Θ 表示。	0.8pt

From now on, we can assume that the value of b is known.

后面的问题，我们可以假定 b 为已知。

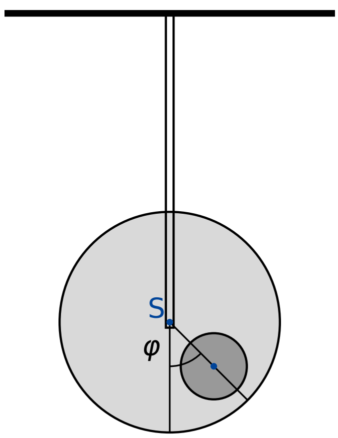

Figure 3: Suspended cylinder.
图 3. 悬挂的圆柱。

Next we want to measure the moment of inertia IS of the cylinder with respect to the symmetry axis S. To this end we suspend the cylinder at its symmetry axis. We then turn it away from its equilibrium position by a small angle , and let it go. See figure 3 for the setup. We find that describes a periodic motion with period T.

接下来我们想确定系统关于 S 轴的转动惯量 I_S。为此 使用硬杆固定 S 轴悬挂圆柱，然后使之相对平衡位置转过一个小角度并释放，如图 3 所示。可以看到系统转角 ϕ 成周期运动，周期为 T。

A.2	What motion does describe? Express the moment of inertia IS of the cylinder around its symmetry axis S in terms of T, b and the known quantities (1). You may assume that we are only disturbing the equilibrium position by a small amount so that is always very small. 给出 ϕ 的运动学方程，指出 ϕ 做何种方式运动? 给出系统关于轴 S 的转动惯量 I_S，用 T, b 和 (1) 中的已知参量表示，假定 ϕ 很小。	0.5pt

From the measurements in questions A.1 and A.2, we now want to determine the geometry and the position of the metal disk inside the cylinder.

通过问题 **A.1** 和 **A.2** 的测量，我们现在进一步确定金属盘的尺寸 r_2 和 w_2 以及在木质圆柱中的位置 d。

A.3	Find an expression for the distance d as a function of band the quantities (1). You may also include r2 and w2 as variables in your expression, as they will be calculated in subtask A.5. 给出 d 的表达式，用 b 和 (1) 中的已知参量表示。表达式中仍可包含变量 r_2 和 w_2，这将在子问题 **A.5** 中进行计算。	0.4pt

A.4	Find an expression for the moment of inertia IS in terms of b and the known quantities (1). You may also include r2 and w2 as variables in your expression, as they will be calculated in subtask A.5. 给出转动惯量 I_S 的表达式，用 b 和 (1) 中已知的参量表示。表达式中仍可包含变量 r_2 和 w_2，这将在子问题 **A.5** 中进行计算。	0.7pt

A.5	Using all the above results, write down an expression for w2 and r2 in terms of b, T and the known quantities (1). You may express w2 as a function of r2. 用以上结果，给出 r_2 和 w_2 的表达式，用 b, T 和 (1) 中已知的参量表示。为书写简单，你可以在 w_2 的表达式中包含 r_2。(建议删去)	1.1pt

B 部分. 旋转的空间站 (6.5 分)

Alice is an astronaut living on a space station. The space station is a gigantic wheel of radius R rotating around its axis, thereby providing artificial gravity to the astronauts. The astronauts live on the inner side of the rim of the wheel. The space station itself is so light that we will neglect its gravitational attraction.

爱丽丝是一个住在空间站的宇航员。空间站是个巨大的半径为 R 的转轮，并关于中轴旋转，由此为宇航员提供了一个假定的重力。宇航员生活在转轮的内边缘。空间站质量很轻，万有引力可忽略；空间站地面的曲率可忽略。

B.1	With what angular frequency ω_{ss} does the space station rotate so that the astronauts experience the same gravity g_E as on the Earth' s surface? 空间站以多大角频率 ω_{ss} 旋转可以使宇航员感受到与在地面相同的重力加速度?	0.5pt

Alice and her astronaut friend Bob have an argument. Bob does not believe that they are in fact living in a space station and claims that they are on Earth. Alice wants to prove to Bob that they are living on a rotating space station by using physics. To this end, she attaches a mass m to a spring with spring constant k and lets it oscillate. The mass oscillates only in the vertical direction, and cannot move in the horizontal direction.

爱丽丝和她的宇航员朋友鲍勃发生争论。鲍勃不相信他们实际生活在空间站，而是生活在地球。爱丽丝想用物理证明给 Bob 他们生活在旋转的空间站。于是，她将质量为 m 的物体挂在弹性系数为 k 的弹簧上，并让其振动。重物只在垂直方向振动，在水平方向上无运动。

B.2	Assuming that on Earth gravity is constant with acceleration g_E, what would be the angular oscillation frequency ω_E that one measures? 在地球上实验，假定重力加速度 g_E 是恒定的，地球上的人可测得上述振动的角频率 ω_E 为多大?	0.2pt

B.3	What angular oscillation frequency ω does Alice measure on the space station? 爱丽丝在空间站 测得的角频率 ω 为多大?	0.6pt

Alice is convinced that her experiment proves that they are on a rotating space station. Bob remains skeptical. He claims that when taking into account the change in gravity above the surface of the Earth, one finds a similar effect. Is he right?

爱丽丝确信她的实验结果能够证明他们生活在一个旋转的空间站。鲍勃仍保持怀疑，他声称当考虑地球表面以外引力随空间的变化，仍可得到类似现象。接下来我们探究为何鲍勃是对的。

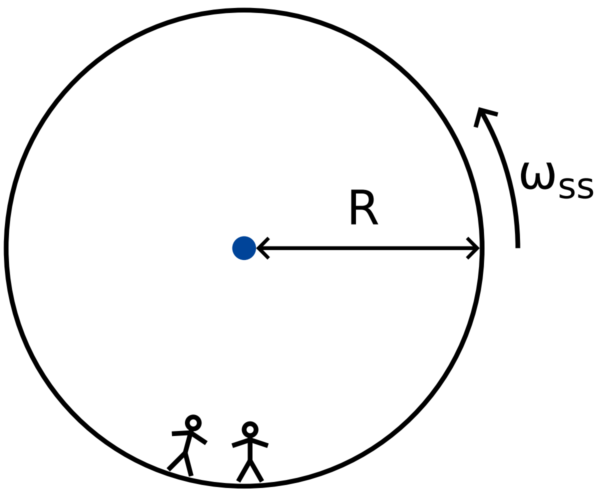

图 4. 空间站

B.4	Derive an expression of the gravity $g_E(h)$ for small heights h above the surface of the Earth and compute the angular frequency $\tilde{\omega}_E$ (linear approximation is enough). The radius of the Earth is given by R_E. 推导地球表面附近高度 h 为小量处重力加速度 $g_E(h)$ 的表达式，并计算振动频率 $\tilde{\omega}_E$ (线性近似足矣)。已知地球半径为 R_E。忽略地球自转。	0.8pt

Indeed, Alice finds that the spring pendulum oscillates with the frequency that Bob predicted.

事实上，爱丽丝确实发现对于该空间站弹簧的振动与 鲍勃所指出的频率一致。

B.5	For what radius R of the space station does the oscillation frequency ω match the oscillation frequency $\tilde{\omega}_E$ on the surface of the Earth? 空间站的半径 R 为多大 ω 和 $\tilde{\omega}_E$ 相一致？结果用 R_E 表示。	0.3pt

Exasperated with Bob’s stubbornness, Alice comes up with the idea of using the Coriolis force to prove her point. To this end she climbs on a tower of height H over the ground of the space station and drops a mass.

被鲍勃的顽固所激怒，爱丽丝想到了一个实验证明她的观点。为此她爬到距空间站地面高为 H 的塔顶并下落了一个物体。这个实验可以理解为在一转动参照系中，也可以理解为在一惯性参照系中。

The Coriolis force is a fictitious force that appears in a uniformly rotating frame of reference. The force $\vec{F}_C$ acting on an object of mass m moving at velocity $\vec{v}$ in a rotating frame with constant angular frequency $\vec{\omega}_{ss}$ is given by

在一匀速转动参照系中，宇航员会感受到一个称作科里奥利力 $\vec{F}_C$ 的假想力。在角速度为 $\vec{\omega}_{ss}$ 转动参照系中，作用于质量为 m，以速度 $\vec{v}$ 运动的物体，科里奥利力 $\vec{F}_C$ 表示为：

$$\vec{F}_C = 2m\vec{v} \times \vec{\omega}_{ss} \, . \tag{2}$$

In terms of the scalar quantities you may use

也可以标量表示为：

$$F_C = 2mv\omega_{ss} \sin\phi \, , \tag{3}$$

where ϕ is the angle between the velocity and the axis of rotation. The force is perpendicular to both the velocity v and the axis of rotation. The sign of the force can be determined from the right-hand rule, but in what follows you may choose it freely.

其中 ϕ 为速度与转轴的夹角，科里奥利力与速度 v 和转轴垂直。力的符号由右手定则确定。but in what follows you may choose it freely.（建议删去不译）

B.6	计算物体落到地面时的水平方向速度 v_x 和位移 d_x（相对于塔基，在垂直于塔的方向上），假定 H 很小，下落过程中加速度不变，并假定 $d_x \ll H$	1.1pt

To get a good result, Alice decides to conduct this experiment from a much taller tower than before. To her surprise, the mass hits the ground at the base of the tower, so that $d = 0$.

为得到一个更好的结果，爱丽丝决定在一个更高的塔实施实验，让她惊奇的是重物落在了塔基处，即 $d_x = 0$。

B.7	计算出发生上述现象（即，$d_x = 0$）塔高的下限。	1.3pt

Alice is willing to make one last attempt at convincing Bob. She wants to use her spring oscillator to show the effect of the Coriolis force. To this end she changes the original setup: She attaches her spring to a ring which can slide freely on a horizontal rod in the x direction without any friction. The spring itself oscillates in the y direction. The rod is parallel to the ground and perpendicular to the axis of rotation of the space station. The xy plane is thus perpendicular to the axis of rotation, with the y direction pointing straight towards the center of rotation of the station.

爱丽丝想为说服鲍勃做最后努力。她想用她的弹簧振子演示科里奥利力的效应。于是她改变了原来的实验装置：她将弹簧挂在一个环上，环可以在沿 x 方向的水平杆上无摩擦自由滑动。弹簧本身在 y 方向振动。杆平行于空间站地面，垂直于空间站转轴。因此 xy 平面也垂直于空间站转轴，y 方向指向空间站转动中心。

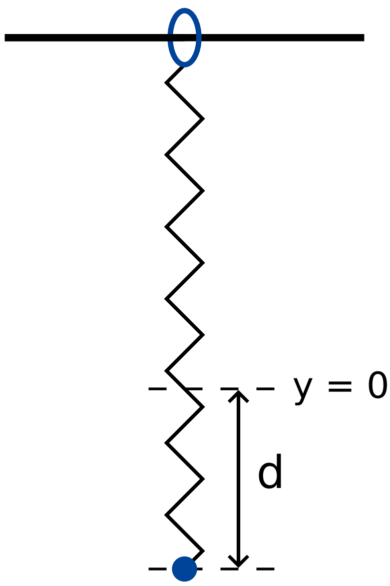

Figure 4: Setup.
图 5. 装置

B.8 Alice pulls the mass a distance d downwards from the equilibrium point $x = 0$, $y = 0$, and then lets it go (see figure 4). **1.7pt**
爱丽丝向下方拉动重物离开平衡点（$x = 0, y = 0$）距离为 d，然后释放（见图 5）。

- Give an algebraic expression of $x(t)$ and $y(t)$. You may assume that $\omega_{ss}d$ is small.
  给出 $x(t)$ 和 $y(t)$ 的代数表达式，可假定 $\omega_{ss}d$ 很小，并忽略 y 方向上运动引起的科里奥利力。
- Sketch the trajectory $(x(t), y(t))$, marking all important features such as amplitude.
  画出运动轨迹 $(x(t), y(t))$。标示出所有重要特征，如振幅等。