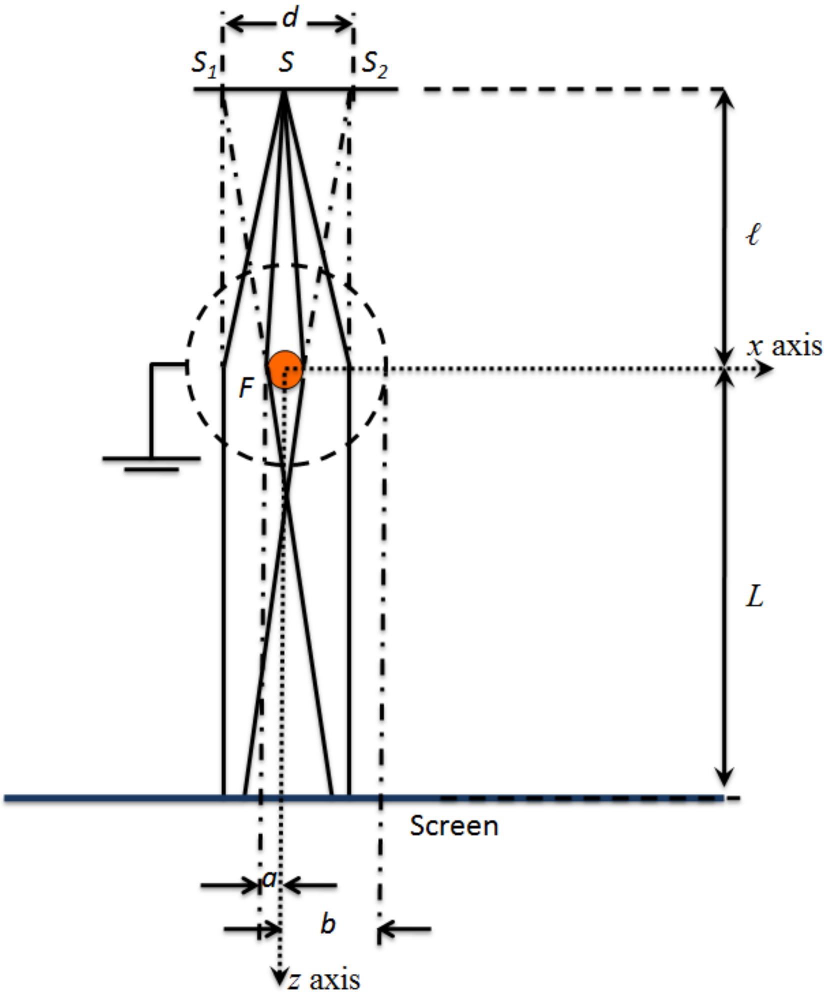
Question 2

The two-slit electron interference experiment was first performed by Möllenstedt *et al*, Merli-Missiroli and Pozzi in 1974 and Tonomura *et al* in 1989. In the two-slit electron interference experiment, a monochromatic electron point source emits particles at S that first passes through an electron "biprism" before impinging on an observational plane; S_1 and S_2 are virtual sources at distance d. In the diagram, the filament is pointing into the page. Note that it is a very thin filament (not drawn to scale in the diagram).



The electron "biprism" consists of a grounded cylindrical wire mesh with a fine filament F at the center. The distance between the source and the "biprism" is ℓ, and the distance between the distance between the "biprism" and the screen is L.

(a) (**2 points**) Taking the center of the circular cross section of the filament as the origin *O*, find the electric potential at any point (x,z) very near the filament in terms of V_a, a and b where V_a is the electric potential of the surface of the filament, a is the radius of the filament and b is the distance between the center of the filament and the cylindrical wire mesh. (Ignore mirror charges.)

(b) (**4 points**) An incoming electron plane wave with wave vector k_z is deflected by the "biprism" due to the x-component of the force exerted on the electron. Determine k_x the x-component of the wave vector due to the "biprism" in terms of the electron charge, e, v_z, V_a, k_z, a and b, where e and v_z are the charge and the z-component of the velocity of the electrons ($k_x \ll k_z$). Note that $\vec{k} = \frac{2\pi\vec{p}}{h}$ where h is the Planck constant.

(c) Before the point *S*, electrons are emitted from a field emission tip and accelerated through a potential V_0. Determine the wavelength of the electron in terms of the (rest) mass m, charge e and V_0,

(i) (**2 points**) assuming relativistic effects can be ignored, and

(ii) (**3 points**) taking relativistic effects into consideration.

(d) In Tonomura *et al* experiment,

v_z = $c/2$,
V_a = 10 V,
V_0 = 50 kV,
a = 0.5 μm,
b = 5 mm,
ℓ = 25 cm,
L = 1.5 m,
h = 6.6×10^{-34} Js,
electron charge, $-e = -1.6 \times 10^{-19}$ C,
mass of electron, $m = 9.1 \times 10^{-31}$ kg,
and the speed of light *in vacuo*, $c = 3 \times 10^{8}$ ms^{-1}

(i) (**2 points**) calculate the value of k_x,

(ii) (**2 points**) determine the fringe separation of the interference pattern on the screen,

(iii) (**1 point**) If the electron wave is a spherical wave instead of a plane wave, is the fringe spacing larger, the same or smaller than the fringe spacing calculated in (ii)?

(iv) (**2 points**) In part (c), determine the percentage error in the wavelength of the electron using non-relativistic approximation.

(v) (**2 points**) Calculate the distance d between the apparent double slits.