Space Debris

Introduction

In more than half a century of space operations quite a large number of man-made objects have been amassed near Earth. The objects that do not serve any particular purpose are called **space debris**. The most attention is usually paid to the larger debris objects, i.e. defunct satellites and spent rocket upper stages, which stay in orbit after delivering their payload. Collisions of such objects with each other may result in thousands of fragments endangering all current space missions.

There is a well-known hypothetical scenario, according to which certain collisions may cause a cascade where each subsequent collision generates more space debris that increase the likelihood of new collisions. Such a chain reaction, resulting in the loss of all near-Earth satellites and making impossible further space programs, is called the **Kessler syndrome**.

To prevent such undesirable outcome special missions are planned to remove large debris object from their present orbits either by tugging them to the Earth's atmosphere or to graveyard orbits. To this end a specially designed spacecraft – a space tug – must capture a debris object. However, before capturing an uncontrolled object it is important to understand its rotational dynamics.

We suggest you to take part in planning of such a mission and find out how the rotational dynamics of a debris object changes in time under the influence of different factors.

Rocket Stage Schematic

The debris object to be considered is a "Kerbodyne 42" rocket upper stage, whose schematic is shown in Fig. 1. The circle line in Fig. 1 marks the outline of a spherical fuel tank.

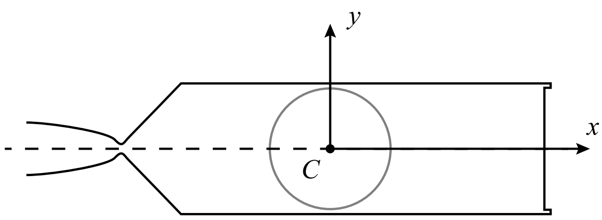

Fig. 1: "Kerbodyne 42" upper stage

We introduce a body-fixed reference frame Cxy with the origin in the center of mass C, x being the symmetry axis of the stage, and y perpendicular to x. The inertia moments with respect to x and y axes are J_x and J_y ($J_x < J_y$).

Part A. Rotation (3.8 points).

Consider an arbitrary initial rotation of the stage with angular momentum L (Fig. 2), where θ is the angle between the symmetry axis and the direction of angular momentum. Fuel tank at this point is assumed to be empty. No forces or torques act upon the stage.

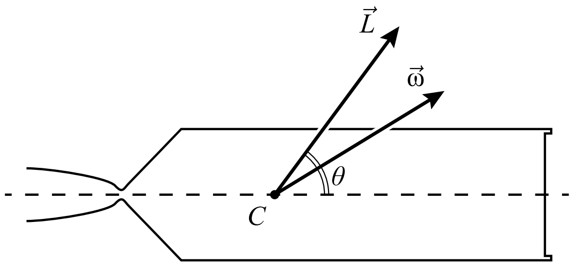

Fig. 2: Rocket stage rotation

A.1	Find the projections of angular velocity $\vec{\omega}$ on x and y, given that $\vec{L} = J_x \omega_x \vec{e}_x + J_y \omega_y \vec{e}_y$ for material symmetry axes x and y with unit vectors $\vec{e}_x$ and $\vec{e}_y$.	0.2pt

A.2	Find the rotational energy E_x associated with rotation ω_x and E_y associated with rotation ω_y. Find total rotational kinetic energy E of the stage as a function of the angular momentum L and $\cos\theta$.	0.4pt

In the following questions of Section A consider the stage's free rotation with the initial angular momentum L and $\theta(0) = \theta_0$.

A.3	Let us denote by x_0 the initial orientation of the stage's symmetry axis Cx with respect to inertial reference frame. Using conservation laws find the maximum angle ψ, which the stage's symmetry axis Cx makes with x_0 during the stage's free rotation. *Note:* Since there are no external torques acting upon the stage, the angular momentum vector remains constant.	1.2pt

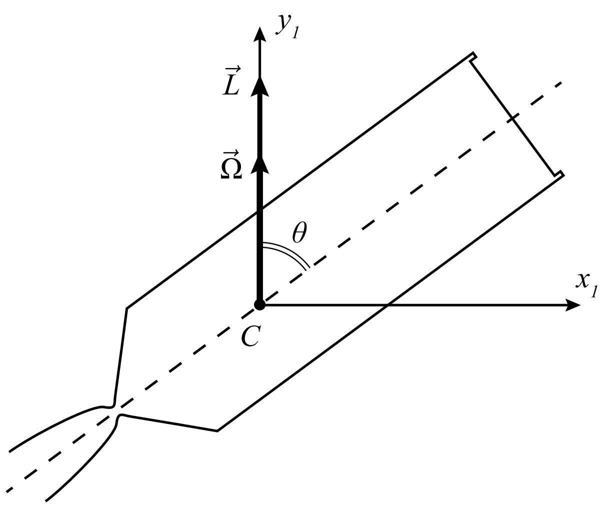

Fig. 3: Precession

Let us now introduce the reference frame $Cx_1y_1z_1$ with y_1 along the constant angular momentum vector $\vec{L}$ (Fig. 3). This reference frame rotates about y_1 in such a way, that the stage's symmetry axis always belongs to the Cx_1y_1 plane.

A.4	Given L and $\theta(0) = \theta_0$, find the angular velocity of the stage $\vec{\omega}_s(t)$ relative to the introduced reference frame and the angular velocity $\Omega(t)$ of the reference frame itself about y_1 as functions of time. Provide the answer for $\vec{\omega}_s(t)$ in the form of projections on Cx_1 and Cy_1. *Note*: angular velocity vectors are additive $\vec{\omega} = \vec{\omega}_x + \vec{\omega}_y = \vec{\Omega} + \vec{\omega}_s$.	2.0pt

Part B. Transient Process (1.6 points).

Most of the propellant is used during the ascent, however, after the payload has been separated from the stage, there still remains some fuel in its tank. Mass m of residual fuel is negligible in comparison to the stage's mass M. Sloshing of the liquid fuel and viscous friction forces in the fuel tank result in energy losses, and after a transient process of irregular dynamics the energy reaches its minimum.

B.1	Find the value θ_2 of angle θ after the transient process for arbitrary initial values of L and $\theta(0) = \theta_1 \in (0, \pi/2)$.	0.6pt

B.2	Calculate the value ω_2 of angular velocity ω after the transient process, given that initial angular velocity $\omega(0) = \omega_1 = 1\, rad/s$ makes an angle of $\gamma(0) = \gamma_1 = 30^\circ$ with the stage's symmetry axis. The moments of inertia are $J_x = 4200\ kg \cdot m^2$ and $J_y = 15\,000\ kg \cdot m^2$.	0.6pt

Part C. Magnetic Field (4.6 points).

Another important factor in rotational dynamics of a debris rocket stage, which is orbiting the Earth, is its interaction with the Earth's magnetic field. Let us first consider an auxiliary problem.

Torque due to Eddy Currents.

Let us place a thin-walled nonmagnetic spherical shell with wall thickness D and radius R in a uniform magnetic field $\vec{B}$, which slowly changes so that its derivative $\dot{\vec{B}}$ is a constant vector making angle α with the direction of $\vec{B}$ (Fig. 4). Electrical resistivity of the shell's material is ρ.

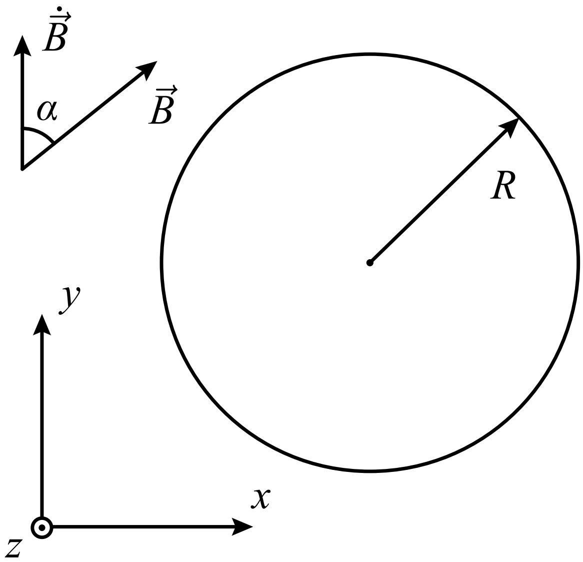

Fig. 4: Spherical shell in magnetic field

C.1	Find the induced magnetic moment $\vec{\mu}$ of the shell, neglecting its self-inductance. Provide the answer for $\vec{\mu}$ in the form of projections on xyz (see Fig. 4).	1.0pt

C.2	Find the torque $\vec{M}$ acting on the spherical shell. Provide the answer for $\vec{M}$ in the form of projections on xyz (see Fig. 4).	0.3pt

Attitude Motion Evolution in the Earth's Magnetic Field

Let us find out how the rotation changes for a rocket stage, which moves in a circular polar orbit with orbital period $T = 100\ min$ (Fig. 5). It transpires that the characteristic times of dynamics due to interaction with the geomagnetic field are much greater than the duration of the transient process. We will now study what happens to the rocket stage after the transient process has completed. To start our analysis consider the stage rotating with angular velocity ω_2 about the axis perpendicular to the orbital plane.

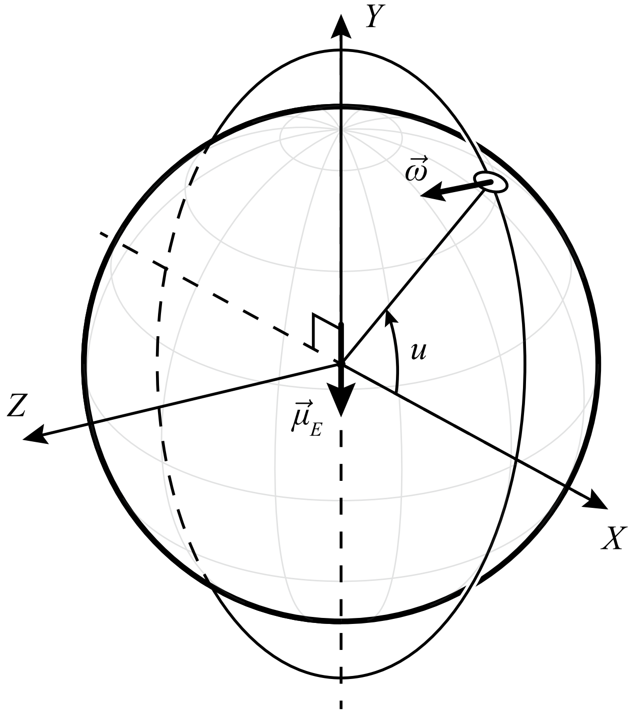

Fig. 5: The orbit

C.3	The Earth's magnetic field $\vec{B}_E$ can be modeled as the magnetic field of a point dipole in the Earth's center. Its dipole moment $\vec{\mu}_E$ is directed opposite to Y axis. The absolute value of the Earth's magnetic field B at the point where the orbit crosses the equatorial plane XZ is $B_0 = 20\ \mu T$. Find $\vec{B}_E(u)$ at a current position of the stage in the orbit defined by the angle u as shown in Fig. 5. The positive direction of u is along with the orbital motion. Provide the answer in the form of the projections of $\vec{B}_E(u)$ on XYZ. *Note*: It may facilitate subsequent calculations if projections of $\vec{B}_E(u)$ are given as functions of $2u$ instead of u. *Note*: Magnetic field of a dipole at point $\vec{r}$ is given by $$\vec{B} = \frac{\mu_0}{4\pi}\left(\frac{3(\vec{\mu},\vec{r})\vec{r}}{r^5} - \frac{\vec{m}u}{r^3}\right). \quad (1)$$	0.4pt

The "Kerbodyne 42" rocket upper stage is mostly made of wood, and the only conductive material is used for its cryogenic fuel tank. We, therefore, consider the stage’s interaction with the geomagnetic field as that of the spherical shell with wall thickness $D = 2\ \mathrm{mm}$, radius $R = 4\ \mathrm{m}$ and resistivity $\rho = 2.7\cdot10^{-8}\ \Omega\cdot\mathrm{m}$.

C.4	Find the torque $\vec{M}(u)$ acting on the stage, as it rotates about the axis perpendicular to the orbital plane with angular velocity ω collinear to Z. Provide the answer in the form of the projections of $\vec{M}(u)$ on XYZ.	1.3pt

C.5	Find the absolute value of angular velocity $\omega(t)$ as a function of time, given that the change in the stage's angular velocity over one orbital period is negligibly small.	1.0pt

C.6	Find the ratio of the orbital period T and the rocket stage's rotation period T_s in the steady-state regime, which sets in after a long time.	1.0pt