**Solutions**

# A single spherical silver nanoparticle

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| 2.1 | Volume of the nanoparticle: .  Mass of nanoparticles:  Number of ions: .  Charge density  Electron concentration , so charge density  Total charge of free electrons ,  Total mass of free electrons . | 0.7 |

**The electric field in a charge-neutral region inside a charged sphere**

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| 2.2 | For a sphere with radius and constant charge density , for any point inside the sphere designated by radius-vector ) Gauss's law yields directly , where is the unit radial vector pointing away from the center of the sphere. Thus, .  Likewise, inside another sphere of radi­us and charge density the field is , where is the radius-vector of the point in the coordinate system with the origin in the center of this sphere.  Merging the two charge configurations gives the setup we want with . So  inside the charge-free region the field is or with the pre-factor | 1.2 |

**The restoring force on the displaced electron cloud**

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| 2.3 | With and we have from above that approximately the filed induced inside the particle is . The number of electrons that produced is negligibly smaller than the number of electrons inside the particle, so (like for a harmonic oscillator).  The work done on the electrons to shift the electron cloud is | 1.0 |

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# The spherical silver nanoparticle in an external constant electric field

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| 2.4 | Inside the metallic particle in the steady state the electric field must be equal to 0. The induced field (from 2.2 or 2.3) compensates the external field: , so .  Charge displaced through the -plane is the total charge of electrons in the cylinder of radius and height : . | 0.6 |

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# The equivalent capacitance and inductance of the silver nanoparticle

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| 2.5a | The electric energy of a capacitor with capacitance holding charges is  . The energy of such capacitor is equal to the work (see 2.3) done to separate the charges (see 2.4), thus . | 0.7 |
| 2.5b | Equivalent scheme for a capacitor reads: . Combining charge from (2.4) and capacitance from (2.5a) gives . | 0.4 |

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| 2.6a | The kinetic energy of the electron cloud is defined as the kinetic energy of one electron multiplied by the number of electrons in the cloud .  The current is the charge of electrons in the cylinder of area and height divided by time , thus . | 0.7 |
| 2.6b | The energy carried by current in the equivalent circuit with inductance is is, in fact, the kinetic energy of electrons . Taking the energy and current from (2.6a) results . | 0.5 |

# The plasmon resonance of the silver nanoparticle

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| 2.7a | From the LC-circuit analogy we can directly derive . Alternatively it is possible to use the harmonic law of motion in (2.3) and get the same result for the frequency | 0.5 |
| 2.7b | rad/s, for light with angular frequency wavelength is . | 0.4 |

# The silver nanoparticle illuminated with light at the plasmon frequency

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| 2.8a | The velocity of an electron . For harmonic motion it is enough to average over period of oscillations. The time-averaged kinetic energy on the electron . During time each electron hits the ions times. So The energy lost in the whole nanoparticle during one period of oscillations is . Time-averaged Joule heating power  .  The expression for current is taken from (2.6a), squared and averaged  . | 1.0 |
| 2.8b | The oscillating current produces the heat in the resistance equal to , what together with results from (2.8a) leads to . | 1.0 |

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| 2.9 | and yields . | 1.0 |

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| 2.10a | Ohm’s law for a *LCR* serious circuit is . At the resonance frequency time-averaged voltage squared is . And from (2.5b) , so Ohm’s law results in . Now time-averaged power loses are and  . | 1.2 |
| 2.10b | Starting with the electric field amplitude , we calculate and . | 0.3 |

# Steam generation by light

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| 2.11a | Total number of nanoparticles in the vessel: Then the total time-averaged Joule heating power: kW. This power goes into the steam generation: , with . Thus the mass of steam produced in second : . | 0.6 |
| 2.11b | The power of light incident on the vessel , and the power directed for steam production by nanoparticles is given in 2.11a. Thus = 0.498. | 0.2 |

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|  | **Total** | **12.0** |